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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

358. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Illinois.

Show that $\dfrac{n(n+1)\ldots(n+m-1)}{m!} - n\dfrac{n(n+1)\ldots(n+m-4)}{(m-3)!} + \dfrac{n(n-1)}{2!}\cdot\dfrac{n(n+1)\ldots(n+m-7)}{(m-6)!} - \ldots = 0$, when $m>2n$; and $=1$ when $m=2n$.

Solution by B. F. FINKEL, Ph. D., Drury College.

We have $(1-x)^{-n}=1+nx+\dfrac{n(n+1)}{1.2}x^2+\dfrac{n(n+1)(n+2)}{1.2.3}x^3+\ldots$

$$+\frac{n(n+1)\ldots(n+m-1)}{m!}x^m+\ldots$$

Also, $(1-x^3)^n=1-nx^3+\dfrac{n(n-1)}{1.2}x^6-\dfrac{n(n-1)(n-2)}{1.2.3}x^9+$

$$\ldots+(-1)^m\frac{n(n-1)(n-2)\ldots.(n+m-1)}{m!}x^{3m}+\ldots$$

The series under consideration is the coefficient of x^m in the product of the right-hand members of these two equations, which is the same as the coefficient of x^m in the product of $(1-x)^{-n}(1-x^3)^n$ or $(1+x+x^2)^n$.

Now the coefficient of x^m in the expansion of $(1+x+x^2)^n$, by the *multinomial theorem*, is

$$\Sigma\,\frac{n!}{\alpha!\,\beta!\,\gamma!},\text{ where }\alpha+\beta+\gamma=n\text{ and }\beta+2\gamma=m.$$

The solutions of these two equations are

$\gamma=0,\ 1,\ 2,\ 3,\ \ldots,\ n;$

$\beta=m,\ m-2,\ m-4,\ m-6,\ \ldots,\ m-2n;$

$\alpha=n-m,\ n-m+1,\ n-m+2,\ n-m+3,\ \ldots,\ 2n-m.$

$$\therefore \frac{n(n+1)(n+2)\dots\dots(n+m-1)}{m!} - n\frac{n(n+1)\dots(n+m-4)}{(m-3)!}$$

$$+\frac{n(n-1)}{1.2.3}\frac{n(n+1)\dots(n+m-7)}{(m-6)!}\dots$$

$$=\frac{n!}{(n-m)!\,m!}+\frac{n!}{(n-m+1)!\,(m-2)!\,1!}+\frac{n!}{(n-m+2)!\,(m-4)!\,2!}+\dots$$

$$+\frac{n!}{(2n-m)!\,(m-2n)!\,n!}.$$

Now, if $m>2n$, there are no terms in the expansion of $(1+x+x^2)^n$ of which the exponent of x is m. Hence, the second member of the above equation is zero. When $m=2n$, there exists only one term of which the exponent of x is $m=2n$, and its coefficient is the last term of the above equation, which is equal to 1.

The fact that there are no terms of which the exponent of x is m when $m>2n$ is manifested also by some of the signs in the factors of the denominators becoming negative.

359. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Illinois.

Show when $1/(1-x)(1-x^3)(1-x^5)(1-x^7)\dots=(1+x)(1+x^2)(1+x^3)(1+x^4)\dots$

Solution by B. F. FINKEL, Ph. D., Drury College.

$$\frac{1}{(1-x)(1-x^3)(1-x^5)\dots(1-x^{2n-1})}$$

$$\equiv\frac{(1+x)(1+x^2)(1+x^3)\dots(1+x^n)}{(1-x)(1+x)(1+x^2)(1-x^3)(1+x^3)(1+x^4)\dots(1-x^{2n-1})}$$

$$\equiv\frac{(1+x)(1+x^2)(1+x^3)\dots(1+x^n)}{(1-x^{2^{p-1}})(1-x^{3.2^{q-1}})(1-x^{5.2^{r-1}})(1-x^{7.2^{s-1}})\dots\dots(1-x^{2n-1})},$$

where $p+q+r+s+\dots=4n-4$,

$$\doteq(1+x)(1+x^2)(1+x^3)\dots(1+x^n)\dots \text{ as } n=\infty,$$

when $-1<x<1$, since, when $-1<x<1$, each factor in the denominator approaches 1.